A Modified Neuber Method Avoiding Artefacts Under Random Loads

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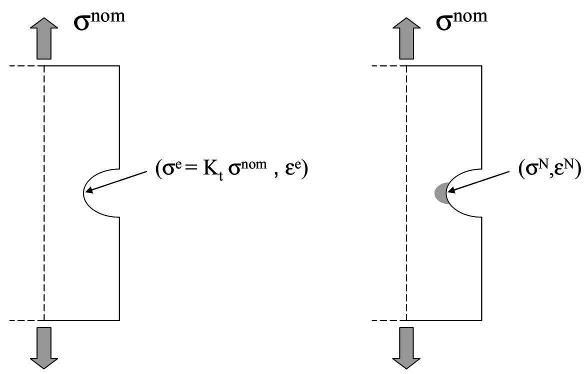
Abstract Surprisingly, Neuber's method is in error when applied to non regular cyclic loads. Some examples are first shown, in which the predicted result by the classical formulation produces a non physical local ratchetting in the notch, for a global load made of a large symmetric cycle and of a small cycle inside the large one. A modification of the original method is then propopsed to avoid the occurence of this artefact. It is successfully tested against finite element computations on several notched specimens.

1 INTRODUCTION

Many engineering components subjected to cyclic loading contain notches like grooves, holes, keyways, welds, etc. When such a component is loaded, a stress concentration appears at the notch root. Fatigue can then be reduced since, even if the component remains globally elastic, a plastic zone develops at the notch root, and an early crack initiation can be observed. In order to predict fatigue life of such components, engineers have to compute the local stresses and strains. Such type of calculation may be quite long, especially when non-linear constitutive equations are used. Simplified methods are then applied, as an alternative to FEM computation. Although many solutions have been investigated, like Molski-Glinka [1], Glinka [2,3], Ellyin and Kujawski [4]… Neuber's rule [5] remains the most currently used for industrial applications. In the case of uniaxial constant amplitude loadings, it appears to be a good approximation; however, a ratchetting phenomenon may appear when the method is applied to variable-amplitude loadings. In this paper, a modification to Neuber's rule is then proposed, and tested for several loadings spectra.

2 PRESENTATION OF NEUBER'S METHOD

This method makes it possible to get an approximation of the elasto-plastic stress and strain at a notch root, from an elastic computation, as represented in Figure 1.



(a) (b)

Figure 1 : Illustration of Neuber's rule
(a) Result of an elastoplastic computation ; (b) local stress and strain according to equation (1)

If this notched component, loaded by a nominal tensile stress σ^{nom}, would remain purely elastic (Figure 1 (a)), this load would induce a local stress σ^e and strain ε^e at the notch root. Defining K_t as the stress concentration factor, σ^e can be expressed as $\sigma^e = K_t\, \sigma^{nom}$. σ^e and ε^e can be reached through a simple elastic FEM computation. To calculate the elasto-plastic stress σ^N and strain ε^N , Neuber [5] postulates that, for a local plasticity, the product of stress and strain at the notch root does not depend on the plastic flow :

$$\sigma^N \varepsilon^N = \sigma^e \varepsilon^e \tag{1}$$

Figure 2 is a graphical representation of Neuber's rule in a stress-strain space. Equation (1) implies that the area of the rectangle defined by σ^e and ε^e is equal to the one defined by σ^N and ε^N. One can also remark that the "elastic point" (σ^e , ε^e) and the "elasto-plastic point" belong to a hyperbola defined by $\sigma\,\varepsilon = \sigma^e\,\varepsilon^e$

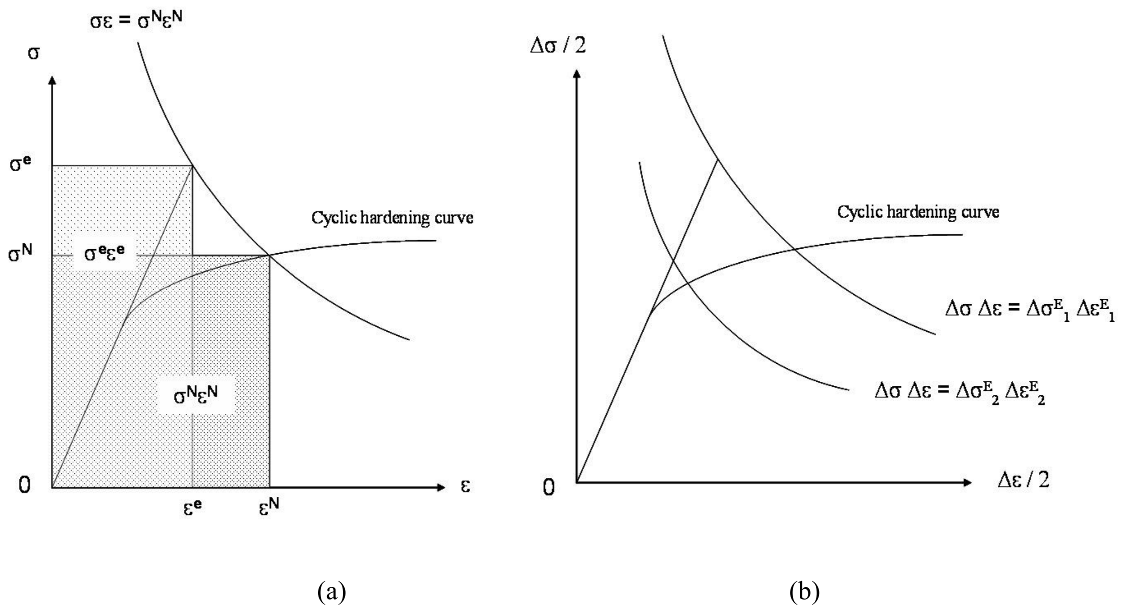


Figure 2 : (a) Classical Neuber's method (b) Classical extension to cyclic loading

This method can be extended to cyclic loadings. The classical approach consists in transferring the classical construction on a cyclic diagram, as illustrated in Figure 2 (b), using stress and strain ranges instead of stress and strain in equation (1). The subsequent point is found on the cyclic curves, following equation (2) :

$$\Delta\sigma^N \Delta\varepsilon^N = \Delta\sigma^e \Delta\varepsilon^e \tag{2}$$

This solution is not satisfactory, since it cannot take into account mean stresses that would be present in global loadings. This is why more precise strategies have been proposed [6,7] (Figure 3), based on an update of the stress-strain curves for each reversal. For a given loading branch (i), the current point always refers to the last couple (σ^N_{i-1} , ε^N_{i-1}) reached in the previous loading step. The resulting computation scheme is given by equation (3). For instance, Figure 3 illustrates the case of the first reversal.

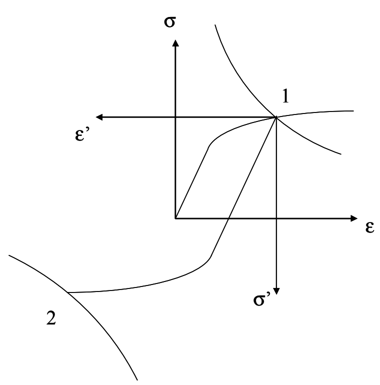


Figure 3 : Extension of Neuber's rule to cyclic loadings with an updating strategy

$$(\sigma_i^N - \sigma_{i-1}^N)(\varepsilon_i^N - \varepsilon_{i-1}^N) = (\sigma_i^e - \sigma_{i-1}^e)(\varepsilon_i^e - \varepsilon_{i-1}^e) \tag{3}$$

Once determined the first maximum stress point 1 by the classical method, one can simply compute the stress and strain for the second extremum 2 by inverting the (σ,ε) axies and choosing point 1 as the new origin.

In some case, the local stress tensor is not uniaxial. A series of possible extensions can also be found in the literature, for instance, using Von Mises invariants instead of components (equation (4))

$$J(\sigma^N)J(\varepsilon^N) = J(\sigma^E)J(\varepsilon^E) \tag{4}$$

This is still an open problem, with no unique solution for plane strain or axisymmetric problems. It will not be treated in this paper, which is restricted to plane-stress cases.

$$J(\sigma^N{}_i - \sigma^N{}_{i-1})J(\varepsilon^N{}_i - \varepsilon^N{}_{i-1}) = J(\sigma^E{}_i - \sigma^E{}_{i-1})J(\varepsilon^E{}_i - \varepsilon^E{}_{i-1}) \tag{5}$$

3 RATCHETTING EFFECT FOR VARIABLE-AMPLITUDE LOADING

A 2D plane-stress notched test specimen is chosen, so that the normal stress is null at the notch root, and an uniaxial stress state appears with only $\sigma_{22} \neq 0$.

When applying Neuber's method to variable amplitude loadings, we observed a ratchetting phenomenon, even for a linear kinematic hardening. Ratchetting was not observed when computing the same notch subjected to the same loading in simulations with the FEM code ZeBuLoN. Moreover, such a localized ratchet is unrealistic in plastic confinement conditions.

Figure 4 illustrates the loading geometry. Only one quarter of the test specimen is modelled, to respect symmetry conditions, displacements are fixed in direction 1 on the specimen axis, and fixed in direction 2 on the bottom. A tensile stress σ^{nom} is applied at the top of the specimen. Figure 5 presents the response at the notch root under the loading of Figure 4 (b). The local elastoplastic stress σ^N is drawn with respect to the local total elastoplastic strain ε^N. On Figure 5 (a) these values are calculated following the classical Neuber algorithm, on Figure 5 (b) they are calculated following the new Neuber algorithm, that will be explained in the next part.

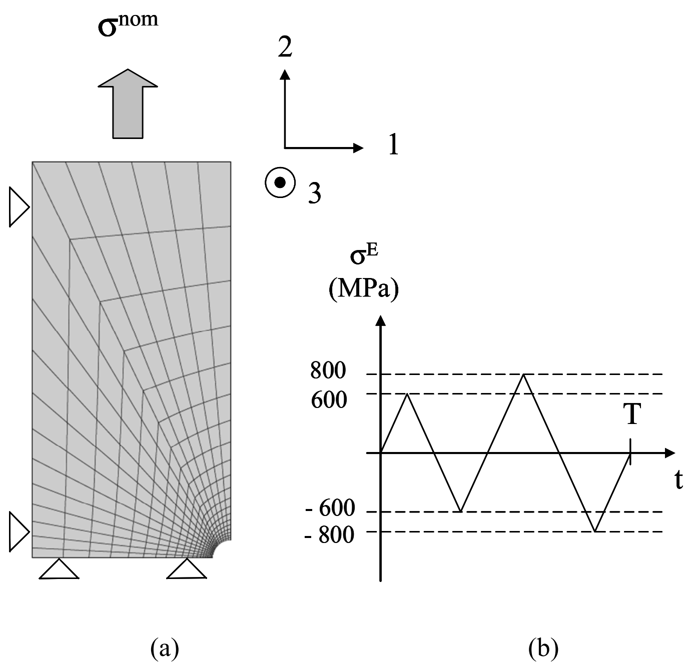
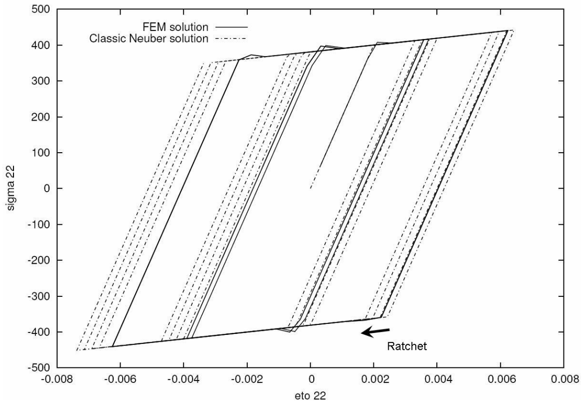


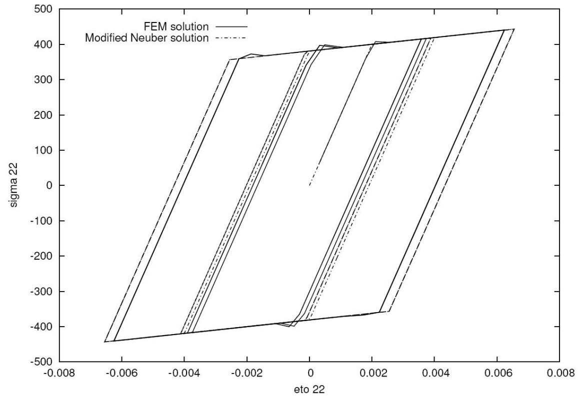
Figure 4 : (a) Mesh and boundary conditions applied to the specimen ; (b) Loading history, $\sigma^e = K_t\, \sigma^{nom}$

Table 1 : Material properties (linear kinematic hardening)

R_0 (MPa)	C (MPa)	d	E (GPa)	ν
400	10000	0	200	0,3



(a)



(b)

Figure 5 : Ratchetting response under the loading defined in Figure 4 (a) Initial algorithm (b) Modified algorithm

4 NEUBER'S METHOD MODIFICATION

Figure 6 shows an elastic loading that will be used to explain the principle of the new strategy. The problem is that the cyclic rule must take into account the point where a "small" loop reaches the "boundary" left by the last large loop. The construction is illustrated in Figure 7. Steps A and B are made according to the previous defined algorithm. The proposed modification is made during step 3: according to classical plasticity, when the reloading branch reaches point C, the subsequent curve must follow the initial branch, with its origin at point O, which produces the evolution after point C shown in step 4'. Keeping B as the origin of the reloading branch would produce an extra-hardening, as shown in step 4.

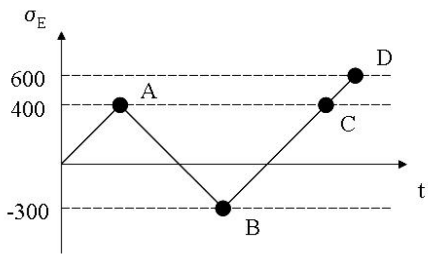
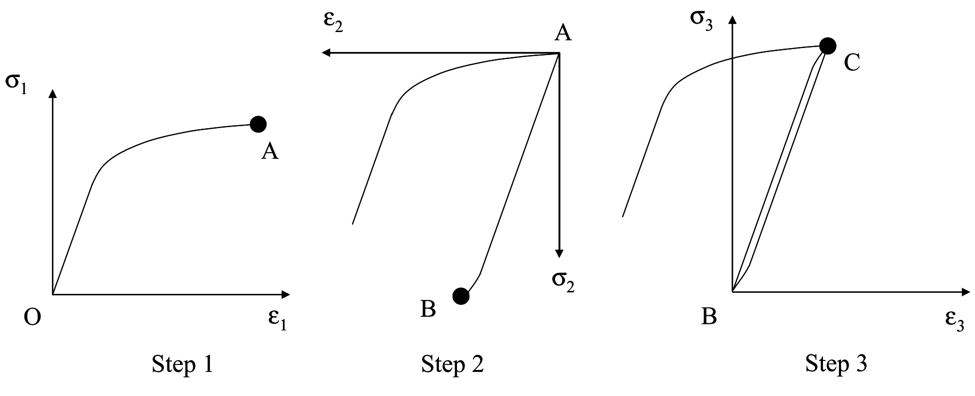


Figure 6 : response of an elastic computation



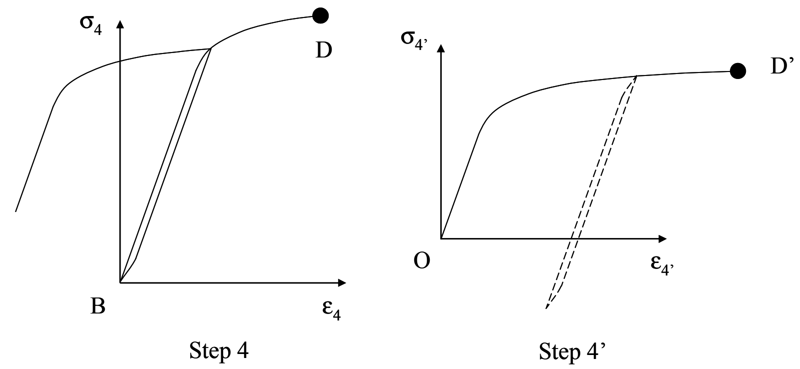


Figure 7 : Neuber's classical (Steps 1, 2, 3, 4) and modified (Steps 1, 2, 3, 4') algorithm application to loading of Figure 6

This algorithm can be compared to the rainflow technique: the classical Neuber method is applied until point C, where $\sigma^E_c = \sigma^E_a$, Once the elasto-plastic loop is closed, it is erased from the loading history, and the elasto-plastic solution for next points like D is calculated by choosing O as origin.

The algorithm is fully detailed in Table 2. It is executed at each stress increment. n is the number of reversals to be erased at the beginning of the algorithm, before computing the solution (σ^N , ε^N). The value of n used for increment i is determined at the end of the algorithm for increment k-1. Pe represents the amplitude of the reversal which ends at point i, according to Neuber's formula. The resulting expression is then: $Pe_i = J(\sigma^E{}_i - \sigma^E{}_{i-1})J(\varepsilon^E{}_i - \varepsilon^E{}_{i-1})$.

Table 2 : The modified algorithm

<u>(*) For each increment k ending at a point i of the loading history:</u>

- Erase the n reversals previous to the current increment from the loading history (n is determined at increment k-1)
- The new origin for Neuber's calculation is now the latest peak before to the erased reversals
- <u>In case of unloading:</u>
  - The new origin for Neuber's calculation is now point i

Re-initialization: n = 0

(**)

- <u>If loading history contains less than 3 reversals:</u> go to (***)
- <u>If</u> $Pe_i > Pe_{i-1-n}$
  - n = n + 2 : the two last reversals will be suppressed at the next increment
  - go to (**)

(***) Calculation of σ^N , ε^N according to Neuber's rule and **go to (*)**

As shown in Figure 5 (b), the present method allows the cycle to remain symmetric for the loading defined in Figure 4 (b). Figure 8 shows a complex history computed with the present algorithm. Reversal suppressions occur at points 1, 2 and 3. The result obtained with a kinematic hardening rule is shown in Figure 9. The material has the same properties than in Table 1. A non linear kinematic hardening rule ($\dot{X} = C\dot{\varepsilon}^p - dX\left|\dot{\varepsilon}^p\right|$, material properties in Table 3) gives the result presented in Figure 10.

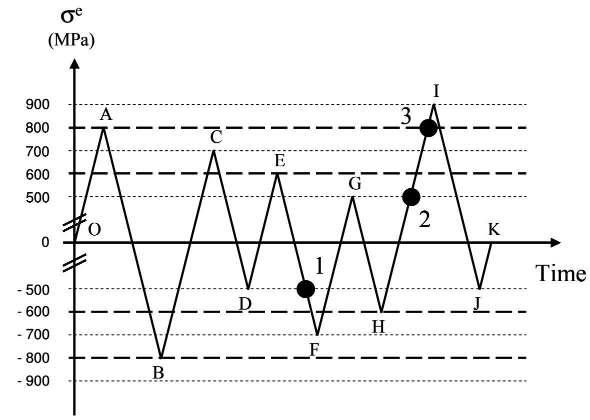
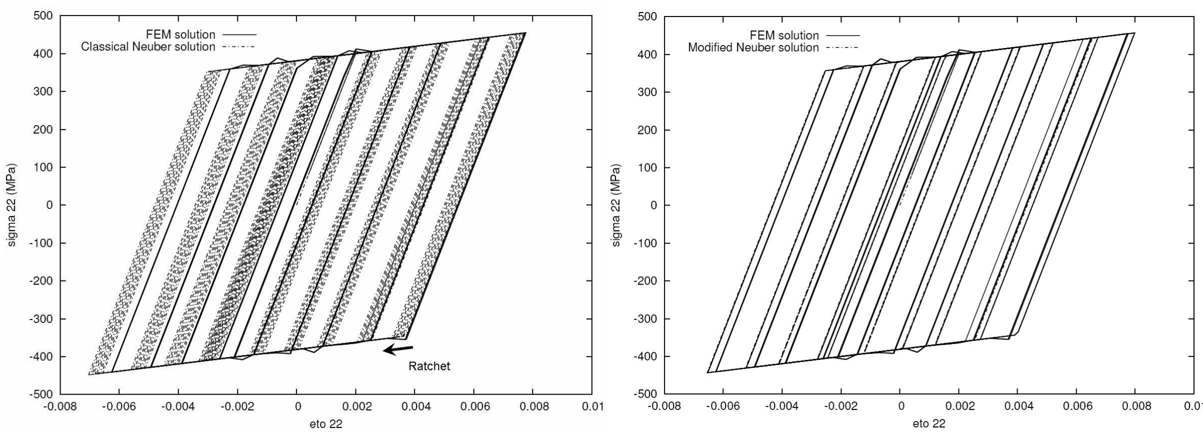


Figure 8 : Test loading history showing asymmetric reversals

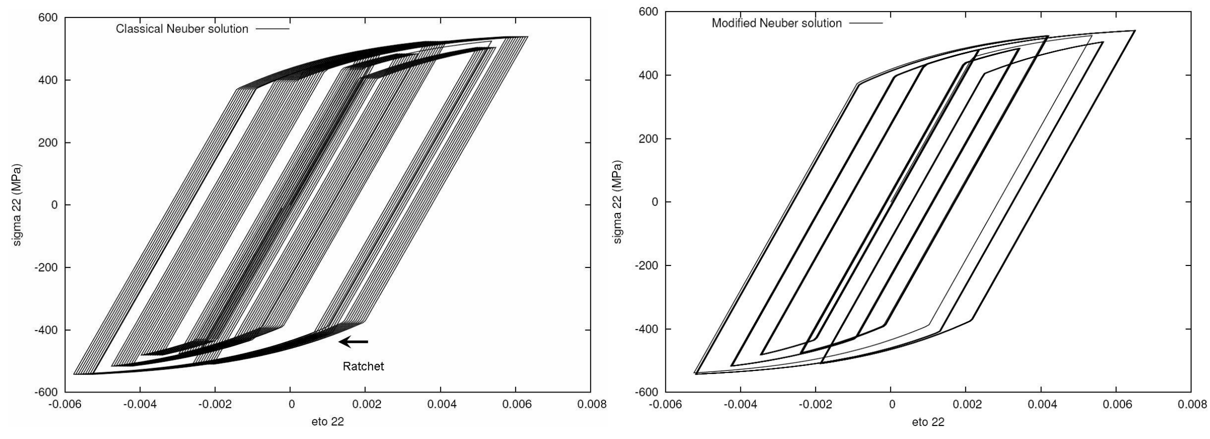


(a) (b)

Figure 9 : Elastoplastic response to loading of Figure 8 with a linear kinematic hardening following : (a) Classical Neuber's algorithm (b) New Neuber algorithm

Table 3 : Material properties (non-linear kinematic hardening)

R_0 (MPa)	C (MPa)	d	E (GPa)	ν
457	40000	400	210	0,3



(a) (b)

Figure 10 : Elastoplastic response to loading of Figure 8 with a non-linear kinematic hardening following: (a) Classical Neuber's algorithm (b) New Neuber algorithm

5 CONCLUSIONS

Local stress and strain history was determined following classical Neuber method at the notch root of a notched component under variable-amplitude loading. The elastoplastic solution was compared to a solution reached by FEM computation. A ratchetting effect that did not appear by using FEM, was observed by using Neuber's rule. Considering that this ratchet was due to asymmetric cycles, a modified Neuber method was proposed. Like the classical one, this new algorithm can only be applied to uniaxial stress states. It is also based on Neuber equivalence rule, but a change of origin at certain moments of the history makes it possible to avoid the shift due to asymmetric cycles. This new algorithm was tested successfully under several complex load spectra, for linear and non-linear kinematic hardening rules.

6 REFERENCES

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